A STATISTICAL MODEL FOR DETERMINING NAVAL ACADEMY INPUTS TO ASSURE SPECIFIED NAVAL OFFICER OUTPUTS

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Submitted in partial fulfillment of
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ABSTRACT

A statistical model for determining Naval Academy inputs and resultant Naval Officer outputs has been developed. Data pertaining to Naval Academy classes for the past thirty years was collected and studied. The basic model considered each graduate sampled as a BERNOULLI Trial. By regression analysis, best estimators for the parameters (proportions of success) for each population of graduates in different grades, were determined. Probabilities were then calculated for a randomly selected graduate, commissioned ENSIGN, to serve in successive grades, with a specified confidence. An INPUT/OUTPUT formula was derived. This paper will provide current personnel planners with accurate officer planning requirements.

The author wishes to express his appreciation for the thoroughly outstanding assistance and encouragement given him by Professor W. M. Woods of the U. S. Naval Postgraduate School.

LIST OF ILLUSTRATIONS

1. Introduction

Naval Academy graduates commissioned in the United States Navy constitute a continuous pipeline of professional cadre for the Naval Service. Each year there is an annual input to the fleet from the U. S. Naval Academy. In addition, annually there is attrition from those Academy graduates on duty in the Navy. This attrition is due to (1) death, (2) normal retirement, (3) voluntary resignation, (4) early medical retirement, (5) discharges and resignations of an involuntary nature.

The Officer Corps of the Navy receives its inputs from several sources in addition to the Naval Academy, since the Naval Academy alone can no longer supply the requirements of a Navy which has over seventy-five thousand officers.

This paper is written with the view towards providing current naval personnel planners with probability statements concerning future INPUTS/OUTPUTS of the Naval Academy. Conclusions developed are considered valid and can assist in future decisions regarding this subject.

Once the Naval Academy graduate who is commissioned Ensign and commences his service, he is simply absorbed in the Officer Corps of the Navy. For many reasons and certainly some of which are due to uncertainties, the Bureau of Naval Personnel periodically will experience shortages (surpluses) in a particular officer grade (and designator). Regardless of the cause, it would seem that improved planning can help in keeping these periodic problems to a minimum. If it is assumed that the Naval Academy has an important mission to perform in providing this special cadre, then it is important to know what OUTPUTS can be expected from specified INPUTS.

Standard statistical methods will be used to develop, from reliable data, inferences and conclusions concerning inputs to the U. S. Naval Academy necessary in order to produce given numbers of officers in the various grades.

Figure 1 represents a FLOW DIAGRAM of the study in the context assumed.

This study could also be used in a related costing analysis, and further is considered relevant to career planning. The study also presents quantitatively an indication of the payoff of the Naval Academy to the Navy. At the outset, it was not possible to foresee final conclusions and inferences.

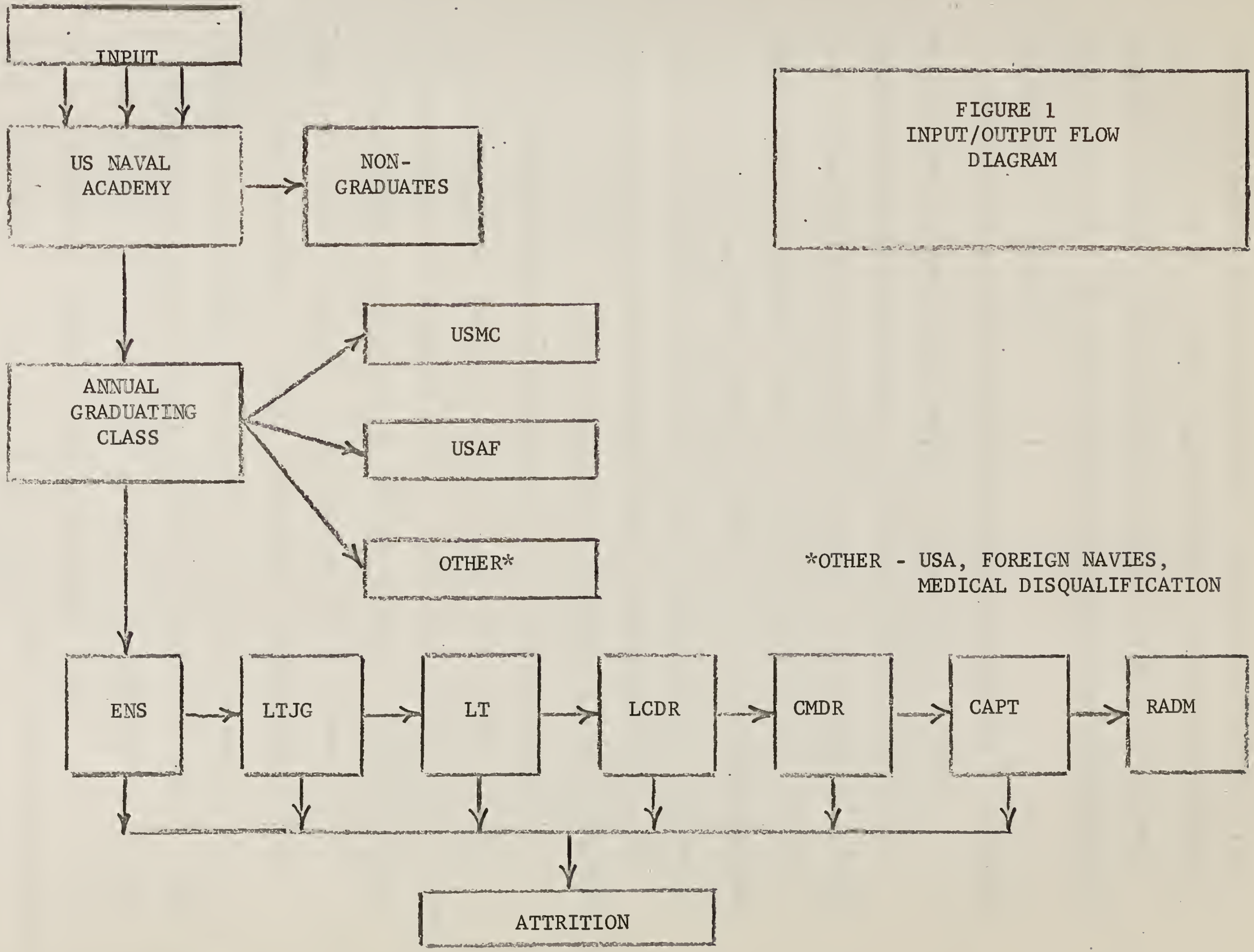


FIGURE 1
INPUT/OUTPUT FLOW DIAGRAM

*OTHER - USA, FOREIGN NAVIES, MEDICAL DISQUALIFICATION

2. Assumptions and Statement of the Problem

By an analysis of the empirical data, probability statements can be made about the future requirements. Naval Academy classes sampled represent a sample of the population of Naval Academy graduates who serve in the Navy. The most significant underlying assumption used in this study is as follows:

Estimators have been determined from data for various U. S. Naval Academy classes which either commenced active duty or underwent a promotion during the past decade. The data used has been obtained from the time period of a cold war/deterrent era and a general economic growth period of the United States. In the event that these two conditions become invalid by unforeseen events, the conclusions developed herein can no longer be considered valid. Lacking further information and considering the range of UNCERTAINTY in any planning study, this assumption does not, in the opinion of this officer, appear unreasonable.

In order to study the population of Naval Academy graduates, certain simplifying conditions have been imposed in order to reduce this population to a manageable size. These are (1) correlation (if any) with Naval Academy class standing, Brigade Aptitude, Athletic achievements, etc., and officer career patterns has not been made; (2) no attempt has been made to include in the study reference to officer designator codes. Numbers of officers in a particular grade include all designator types; (3) no analysis has been made of graduates "lost" to the other Armed Services, although data is included for information; (4) underlying each progressive grade for a year group is the selection board opportunity. An analysis of this opportunity as such has not been made; i.e., no direct inferences can be made on the percentage of

officers in a particular class who failed selection; (5) lastly, no attempt has been made to quantitatively reflect the influence of promotion requirements for service in grade. A four-year Ensign is equivalent to a one and one-half year Ensign. These service requirements, based in some degree on the needs of the service, now appear (and have been) quite stable.

The statement of the Problem involves a method whereby reliability statements concerning the output of the U. S. Naval Academy to the Navy can be made. The following questions will be answered:

a. How large a fourth class is necessary in order to assure the Navy of at least 100 officers of a specified rank; for example, Lieutenant Commander with probability .80? With probability .95?

b. What is the probability that a graduate commissioned in the Navy, will serve in a specified rank?

This paper will provide current Naval Officer personnel planners with accurate Naval Academy INPUT/OUTPUT information.

3. Conclusions

The point estimate for the probability of a randomly selected midshipman, fourth class, upon entering the U. S. Naval Academy to graduate and be commissioned Ensign, U. S. Navy, is .590.

A lower confidence limit table for the probability of a U. S. Naval Academy graduate who is commissioned Ensign, U. S. Navy, to serve in future grades is as follows:

TABLE I
LOWER CONFIDENCE LIMITS FOR FUTURE GRADES

FUTURE GRADE	CONFIDENCE LEVELS 95	90	85	80
LTJG	.989	.989	.990	.990
LT	.886	.888	.889	.891
LCDR	.614	.617	.619	.621
CDR	.444	.446	.448	.450
CAPT	.262	.264	.265	.266
RADM (above)	.0247	.0325	.0380	.0422

Example: The probability of a randomly selected Naval Academy graduate commissioned Ensign, U. S. Navy upon graduation will serve in the grade of LCDR is at least .614, with 95% confidence.

Table II is an INPUT/OUTPUT table for current officer planning in the Navy. It gives fourth class inputs necessary for fixed outputs of naval officers in different grades.

TABLE II

U. S. NAVAL ACADEMY INPUTS NEEDED FOR FIXED OUTPUTS

	USNA OUTPUTS						
USNA INPUTS	ENS	LTJG	LT	LCDR	CDR	CAPT	RADM
171	101	←100→	90	62	45	26	2
191	113	111	←100→	69	50	29	3
276	163	161	144	←100→	72	43	4
382	225	222	200	138	←100→	59	6
646	382	377	339	234	170	←100→	9

Example: At least 382 fourth classmen are necessary in order to "produce" 100 Naval Officers of grade Commander, which in turn, will "produce" at least 59 Captains, etc., with 95% confidence.

Figure 2 is a graphic representation of raw historical data. Data for the plots in Figure 2 is listed in Table V, Section 4. It is interesting to peruse the different percentages afforded the classes, for which data was collected, for the same rank. For example:(1) 84% of USNA class 1934 ENSIGNS served in the grade of CDR, while only 35% of USNA class 1947 ENSIGNS served in grade of CDR. (2) 85% of USNA class 1935 ENSIGNS served in grade of LCDR, while only 58% of USNA class of 1950 ENSIGNS served in grade of LCDR. Underlying this graphic picture are the impacts upon the Officer Corps of World War II, the economic climate of the United States, the different minimum service requirements

before acceptance of voluntary resignations, and other variable factors which will be discussed in following sections.

Figure 3 is an additional graphic representation of raw historical data. Data for these plots is contained in Table VI, Section 4. These plots show aggregate attrition in grade. For example: (1) 91% of USNA class 1935 LCDR's served in the grade of CDR, whereas only 75% of USNA class 1942 LCDR's served in the grade of CDR. This is further explained in the context that 25% of Class 1942 LCDR's did not serve in grade of CDR. This 25% attrition total is composed of attrition due to non-selection for CDR, death, resignation, and retirement.

The nature of these conclusions suggests that further extensions could be made relating to the costs and effectiveness of the entire PIPELINE system. Marginal costs, retention rates, an optimal size of the fourth class, and more detailed attrition analysis could be further developed.

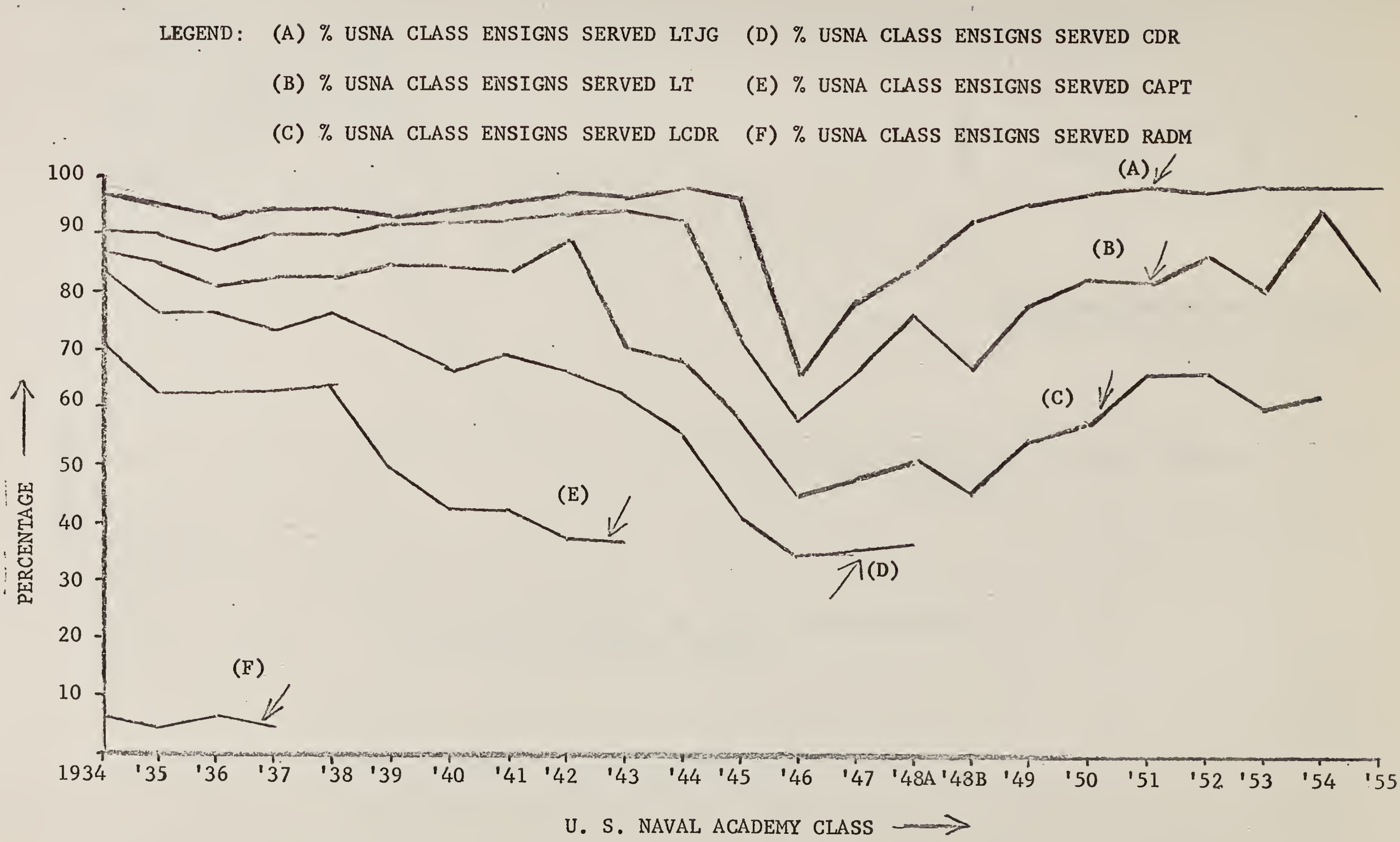
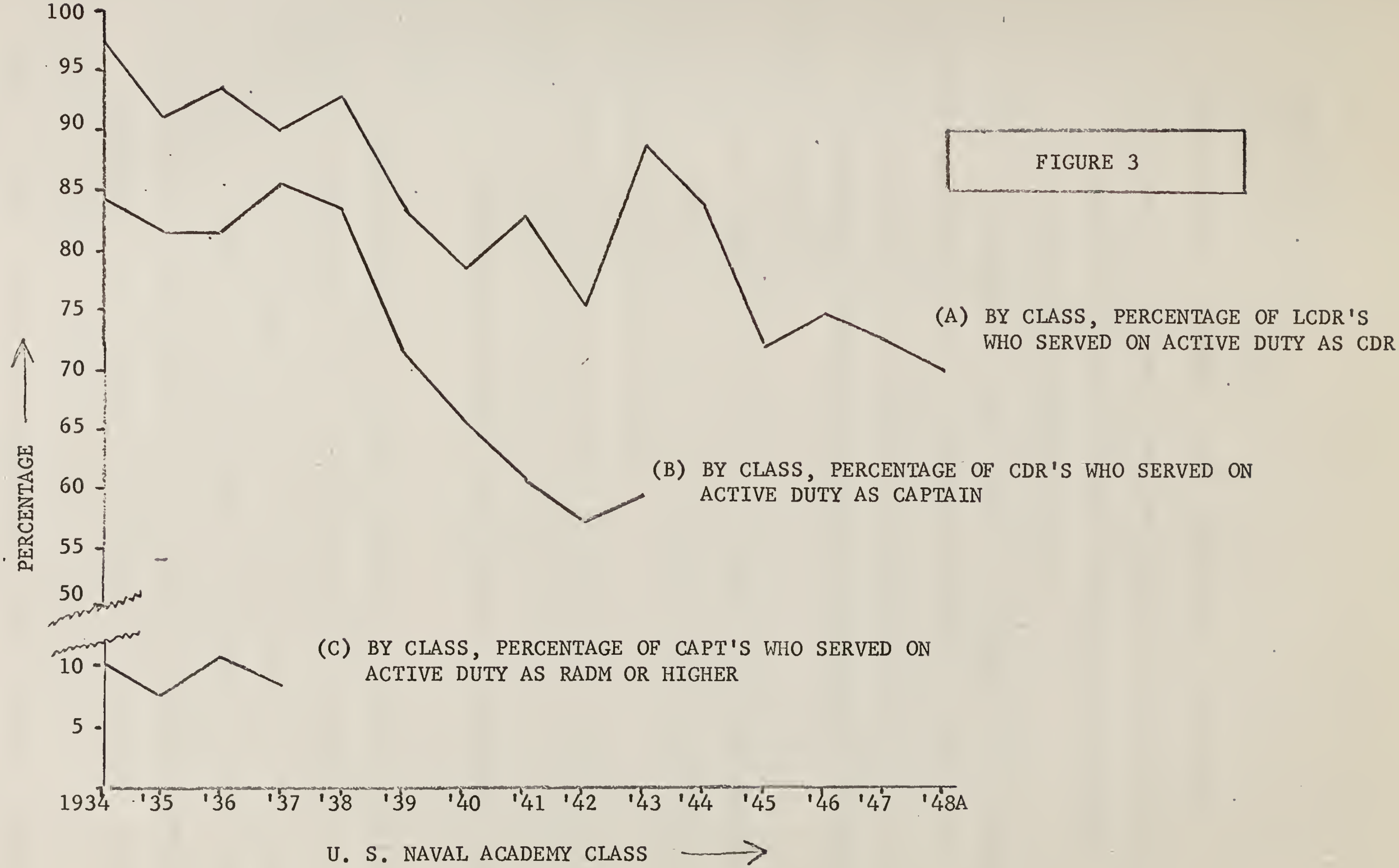

LEGEND: (A) % USNA CLASS ENSIGNS SERVED LTJG (D) % USNA CLASS ENSIGNS SERVED CDR
(B) % USNA CLASS ENSIGNS SERVED LT (E) % USNA CLASS ENSIGNS SERVED CAPT
(C) % USNA CLASS ENSIGNS SERVED LCDR (F) % USNA CLASS ENSIGNS SERVED RADM
PERCENTAGE
100
90
80
70
60
50
40
30
20
10
(A)
(B)
(C)
(D)
(E)
(F)
1934 '35 '36 '37 '38 '39 '40 '41 '42 '43 '44 '45 '46 '47 '48A '48B '49 '50 '51 '52 '53 '54 '55
U. S. NAVAL ACADEMY CLASS

FIGURE 2

FIGURE 3



100
95
90
85
80
75
70
65
60
55
50
10
5
PERCENTAGE
(A) BY CLASS, PERCENTAGE OF LCDR'S WHO SERVED ON ACTIVE DUTY AS CDR
(B) BY CLASS, PERCENTAGE OF CDR'S WHO SERVED ON ACTIVE DUTY AS CAPTAIN
(C) BY CLASS, PERCENTAGE OF CAPT'S WHO SERVED ON ACTIVE DUTY AS RADM OR HIGHER
1934 '35 '36 '37 '38 '39 '40 '41 '42 '43 '44 '45 '46 '47 '48A
U. S. NAVAL ACADEMY CLASS

4. Compilation of Data

Source of data used were:

Ref a) Register of ALUMNI - 1964, USNA Alumni Association

Ref b) Register of Commissioned and Warrant Officers of U. S. Navy and Marine Corps and Reserve Officers on Active Duty 1 January 1964 (NAVPERS 15,018)

Ref c) Current List of Flag Officers, including FY 1965 selectees

Data was collected during the period October 1964 - January 1965. Size of graduating classes and numbers of non-graduates are listed in ref (a).

Determination of numbers of graduates commissioned in the Navy was made from ref (a).

Data was compiled for U. S. Naval Academy class 1934-1963 inclusive. No effort was made to collect data on earlier classes due to the vast environmental differences involved.

Reference (b) was used to verify reference (a) in regard to the numbers of graduates serving in a particular grade, for which that particular year group had within three years been considered for promotion.

No attempt was made to determine numbers in a class for a rank in which the time period elapsed since first becoming eligible for promotion was not sufficient to rule out further promotion. For example, the class of 1950 first up for Commander in calendar year 1964 could only be used as a sample population for rank of Lieutenant Commander.

Biographical data contained in reference (a) was considered quite accurate in general. Reference (b) as stated was used to verify "highest ranks" in cases where reference (a) was considered inaccurate.

There is a very small percentage of Naval Academy graduates who

subsequently resign from the Navy after serving on active duty, and who then re-enter the Navy either voluntarily or involuntarily. An officer in this category was "counted" as a resignation in the grade in which he resigned.

Basic data is contained in Tables III and IV. Tables V and VI were computed from Table III. Table VII is a combination of raw data and percentages pertaining to U. S. Naval Academy inputs and outputs.

TABLE III

NAVAL OFFICER OUTPUTS FROM NAVAL ACADEMY CLASS (YEAR)

USNA CLASS	NR ENSIGNS	NR LTJG	NR LT	NR LCDR	NR CDR	NR CAPT	NR RADM
1934	330	319	302	285	277	234	24
1935	369	352	333	314	286	234	18
1936	219	204	192	179	168	137	15
1937	266	251	239	219	197	169	15
1938	386	365	347	319	296	247	
1939	504	472	462	432	362	251	
1940	396	374	366	335	262	172	
1941	352	336	328	296	245	149	
1942	504	492	473	447	336	192	
1943	574	556	545	408	362	216	
1944	694	682	649	473	395		
1945	852	823	622	497	355		
1946	993	654	585	456	341		
1947	760	598	512	364	265		
1948A	453	381	347	234	164		
1948B	373	346	253	172			
1949	675	653	528	370			
1950	462	454	385	268			
1951	483	477	398	320			
1952	514	507	448	340			
1953	619	616	508	374			
1954	559	552	528	350			
1955	495	489	404				
1956	454	449	375				
1957	567	567	529				
1958	624	617	573				
1959	638	635					
1960	648	644					
1961	664	663					
1962	624						
1963	722						

TABLE IV

NAVAL ACADEMY CLASS DATA

USNA CLASS	NR NON-GRADUATES	NR GRADUATES	NR USMC	NR USAF	NR OTHER
1934	136	463	26		107
1935	132	442	27		46
1936	65	262	25		18
1937	94	323	26		31
1938	122	438	26		26
1939	220	581	24		53
1940	259	456	26		34
1941	160	399	26		21
1942	177	563	24		35
1943	124	616	29		13
1944	139	766	25		47
1945	121	914	29		33
1946	165	1046	36		17
1947	126	820	37		23
1948A	0	500	31		16
1948B	168	410	24	2	11
1949	316	790	55	56	4
1950	178	692	47	169	14
1951	145	725	48	177	17
1952	197	783	63	195	11
1953	171	924	66	226	13
1954	254	855	64	219	13
1955	324	742	56	184	7
1956	352	681	51	168	8
1957	295	848	62	204	15
1958	308	899	69	184	22
1959	303	799	59	83	19
1960	278	797	62	61	26
1961	340	786	59	47	16
1962	285	789	58	82	25
1963	322	871	65	54	30

*OTHER - includes those graduates

(1) disqualified for commission due medical reasons,

(2) commissioned U. S. Army or Foreign Navies,

(3) due billet limitations, did not go on active duty, i.e., USNR

TABLE V

PERCENTAGES OF ENSIGNS WHO SERVED IN
SUCCESSIVELY HIGHER GRADES, BY CLASS

USNA CLASS	# COMMISSIONED ENSIGN	% LTJG	% LT	% LCDR	% CDR	% CAPT	% ADM
1934	330	96.6	91.6	86.4	84	71	7.28
1935	369	95.5	90.4	85	77.5	63.4	4.87
1936	219	93.2	87.6	81.7	76.7	62.6	6.84
1937	266	94.4	90.0	82.4	74	63.6	5.63
1938	386	94.5	90.0	82.6	76.7	64	
1939	504	93.5	92	85.9	72	49.9	
1940	396	94.2	92.4	84.5	66.2	43.4	
1941	352	95.5	93.2	84	69.6	42.4	
1942	504	97.9	94	88.9	66.8	38	
1943	574	97	95	70.4	63.1	37.6	
1944	694	98.5	93.6	68.2	57		
1945	852	96.6	73.1	58.4	41.6		
1946	993	65.7	58.9	46	34.5		
1947	760	78.6	67.6	48	34.8		
1948A	453	84	76.6	51.6	36.2		
1948B	373	92.9	67.9	46			
1949	675	96.7	78.2	54.9			
1950	462	98	83.3	58			
1951	483	99	82.5	66.4			
1952	514	98	87	66.1			
1953	619	99.4	81.5	60.4			
1954	559	99	94.5	62.6			
1955	495	98.5	81.5				
1956	454	98.5	82.5				
1957	567	100	93				
1958	624	99	92				
1959	638	99					
1960	648	99					
1961	664	99					
1962	624						
1963	722						

TABLE VI

PERCENTAGES OF OFFICERS WHO SERVED IN SUCCESSIVELY HIGHER GRADES, BY CLASS

USNA CLASS	(%) LTJG/ENS	(%) LT/LTJG	(%) LCDR/LT	(%) CDR/LCDR	(%) CAPT/CDR	(%) ADM/CAPT
1934	96.6	94.6	94.3	97.2	84.4	10.25
1935	95.5	94.5	94.3	91.0	81.8	7.70
1936	93.2	93.5	93.1	93.7	81.6	10.92
1937	94.4	95.2	91.6	90.0	85.6	8.88
1938	94.5	95.0	92.0	92.9	83.5	
1939	93.5	97.8	93.6	83.6	71.4	
1940	94.2	97.8	91.6	78.3	65.6	
1941	95.5	97.6	90.4	82.7	60.7	
1942	97.9	95.9	94.5	75.0	57.1	
1943	97.0	98.0	75.0	88.6	59.6	
1944	78.5	95.2	73.0	83.5		
1945	96.6	75.7	79.9	71.4		
1946	65.7	89.5	78.1	74.6		
1947	78.6	85.7	71.1	72.7		
1948A	84.0	91.1	67.5	70.1		
1948B	92.9	73.2	68.0			
1949	96.7	80.9	70.1			
1950	98.0	85.0	69.6			
1951	99.0	83.5	80.4			
1952	90.0	88.5	76.0			
1953	99.4	82.4	73.6			
1954	99.0	95.6	66.4			
1955	98.5	82.6				
1956	98.5	83.6				
1957	100	93.1				
1958	99	92.9				
1959	99					
1960	99					
1961	99					
1962						
1963						

TABLE VII

NAVAL ACADEMY ATTRITION BY CLASS

USNA CLASS	NR ENTERED	NR GRADUATED	(%) GRADUATE ENTERED	NR ENSIGNS	(%) ENSIGNS ENTERED
1934	599	463	.774	330	55
1935	574	442	.770	369	64.3
1936	327	262	.802	219	67
1937	417	323	.774	266	63.6
1938	560	438	.783	386	69
1939	801	581	.726	504	62.9
1940	715	456	.640	396	55.5
1941	559	399	.715	352	63
1942	740	563	.760	504	68
1943	740	616	.835	574	77.5
1944	905	766	.848	694	76.6
1945	1035	914	.881	852	82.4
1946	1211	1046	.864	993	81.6
1947	946	820	.868	760	80.4
1948*	1078*	910*	.845*	826*	76.8*
1949	1106	790	.715	675	61
1950	870	692	.795	462	53.2
1951	870	725	.833	483	55.5
1952	980	783	.800	514	52.4
1953	1095	924	.841	619	56.4
1954	1109	855	.774	559	50.5
1955	1066	742	.696	495	46.4
1956	1033	681	.660	454	43.9
1957	1143	848	.742	567	49.5
1958	1207	899	.744	624	51.6
1959	1102	799	.725	638	57.7
1960	1075	797	.742	648	60
1961	1126	786	.700	664	59
1962	1074	789	.733	624	58
1963	1193	871	.730	722	60.5

*USNA Classes 1948A and 48B considered as one class for this table

5. Statistical Analysis

Early analysis of the data revealed vast differences in the number of officer graduates per class serving in the various grades over the period 1934-1963. The data suggested three distinct periods in which the population of graduates was nearly alike. These periods were pre-World War II, Worls War II and the immediate years following, and the cold war era. Figures 4 - 6 reflect many different underlying factors which affected the population proportions. Among these factors are: (1) economic climate of United States, (2) accelerated promotion during World War II, (3) demobilization of the Navy after World War II, (4) twenty-five percent quota of U. S. Naval Academy graduates to the Air Force in the 1950's, (5) variable promotion opportunity after World War II caused by the HUMP, (6) different promotion policies for years in grade, (7) different regulations requiring initial service upon graduation before voluntary resignation. As an example, consider the range for the proportion of $\left(\frac{\text{ENSIGNS}}{\text{NR ENTERED}}\right)$ per class shown in Figure 4. From .82 for class '45 to .44 for class '56! Also, in Figure 5, it is interesting to observe that the distance between plots increases with time.

Figure 6 is a comparison of the three largest and three smallest classes. It simply reveals the stable conditions of the pre-World War II classes (smallest) and vast fluctuations of the largest classes from a different time period.

Ample and reliable data was available but how could meaningful probability statements be determined? Initially, each graduate was thought to be a trial or sample. If he served in the grade under observation, the trial was a success, if not - a failure. Empirical

data for the recent classes could be used provided that the classes sampled were evaluated to be from the same population.

In addition, it would be necessary to assume that conditions which affected the population sampled, such as promotion policy, cold war posture, economic climate, etc., would generally remain the same. That is to say, that the conclusions developed do not foresee a depression, a nuclear exchange, radical changes in promotion or Naval Academy policy.

Figure 2 was considered the key to the analysis and indicated a regression analysis. However, a trend line covering the entire thirty year period did not seem appropriate for estimation of the future, for reasons already explained.

Development of the statistical analysis * follows with specific computations contained in Appendix I.

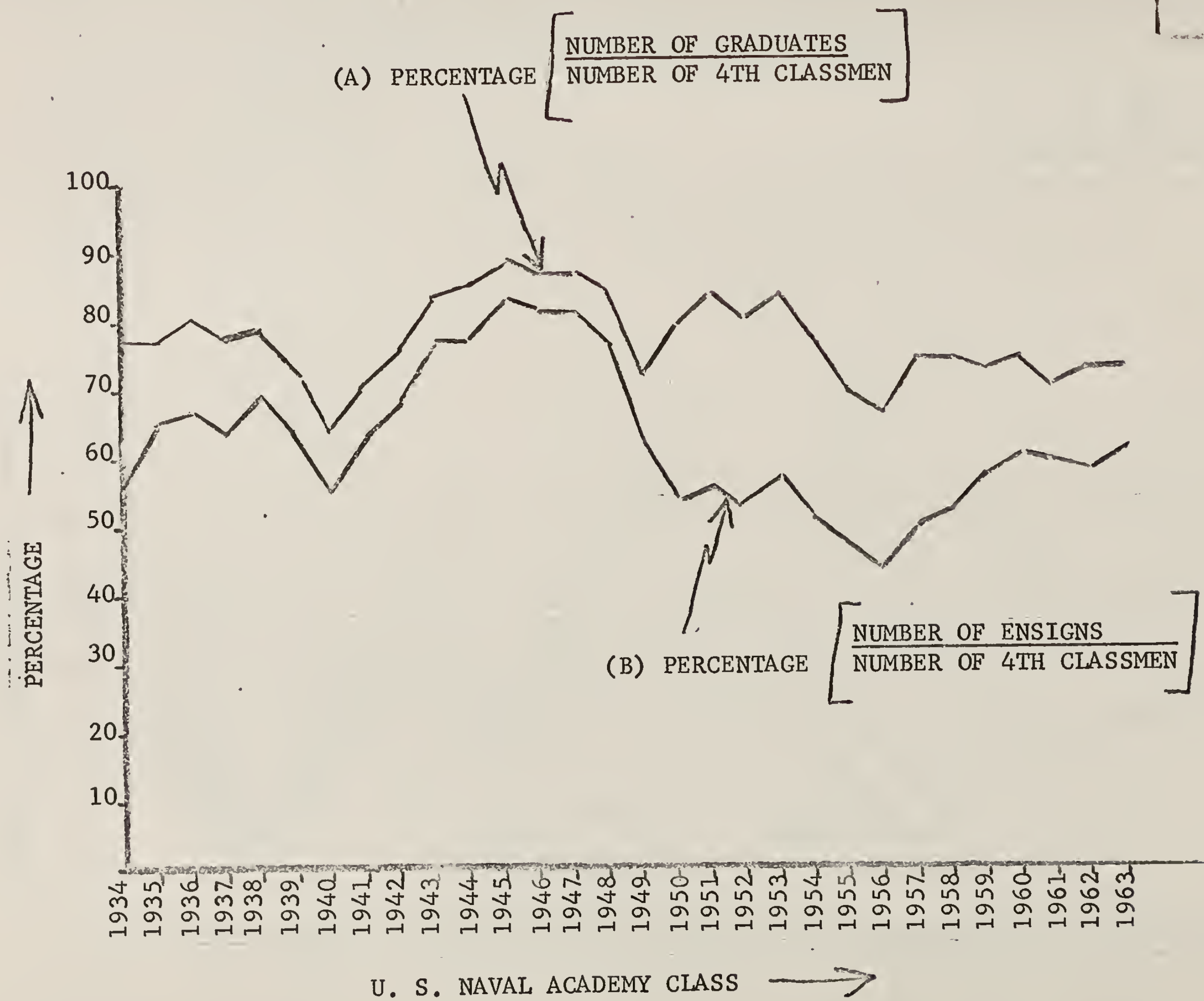
*Standard textbooks used throughout this study are listed for reference:

a. Modern Probability Theory and Its Applications, E. Parzen
John Wiley and Sons, Inc., 1960

b. Introduction to Mathematical Statistics, P. Hoel
John Wiley and Sons, Inc., 1962

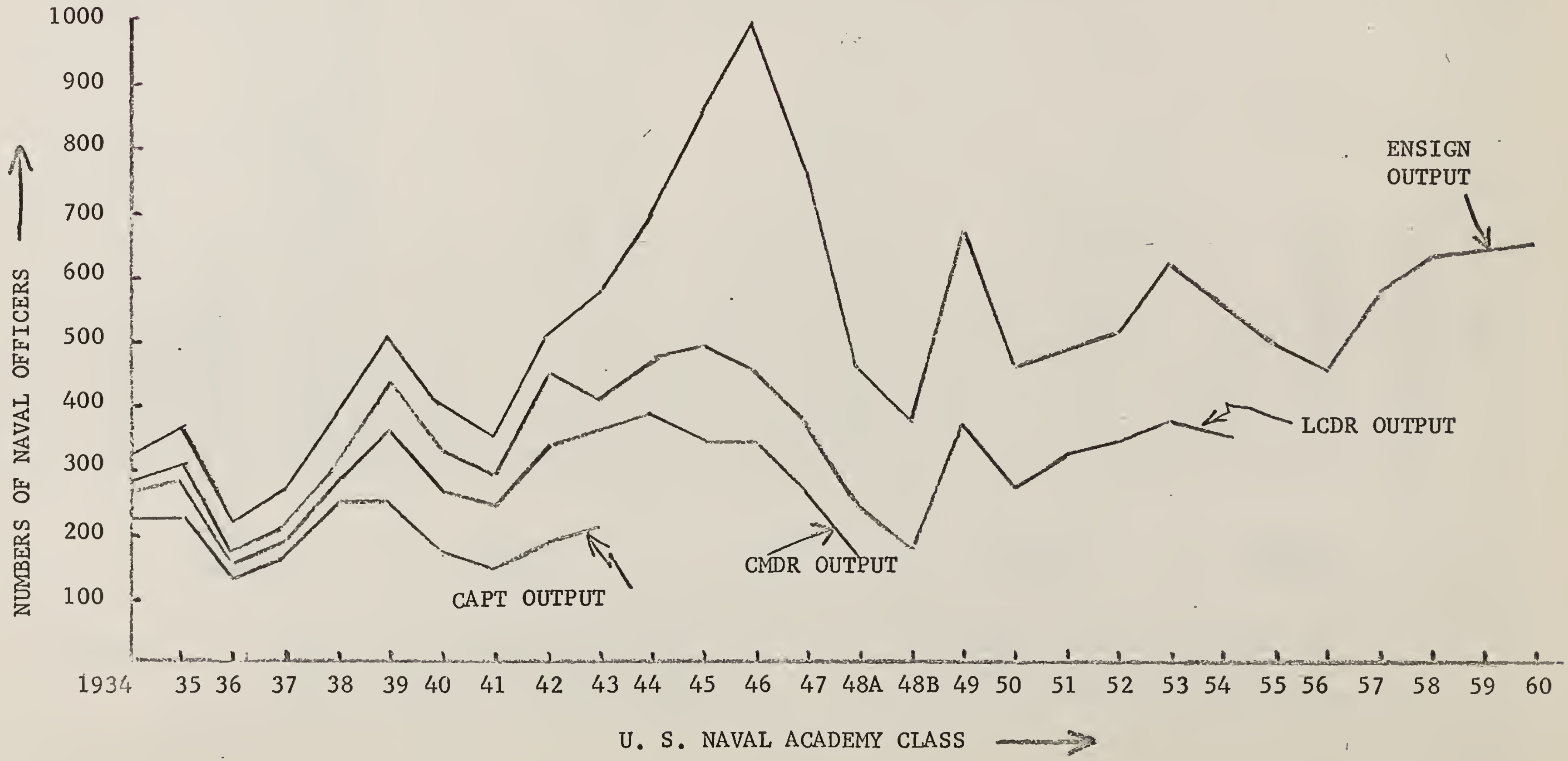
c. Reliability: Management, Methods, and Mathematics, Lloyd & Lipow
Prentice - Hall, Inc., 1962

FIGURE 4



(A) PERCENTAGE [NUMBER OF GRADUATES / NUMBER OF 4TH CLASSMEN]
(B) PERCENTAGE [NUMBER OF ENSIGNS / NUMBER OF 4TH CLASSMEN]
PERCENTAGE
100
90
80
70
60
50
40
30
20
10
1934
1935
1936
1937
1938
1939
1940
1941
1942
1943
1944
1945
1946
1947
1948
1949
1950
1951
1952
1953
1954
1955
1956
1957
1958
1959
1960
1961
1962
1963
U. S. NAVAL ACADEMY CLASS

FIGURE 5
OUTPUTS OF ENS, LCDR, CMDR,
AND CAPT'S BY CLASS



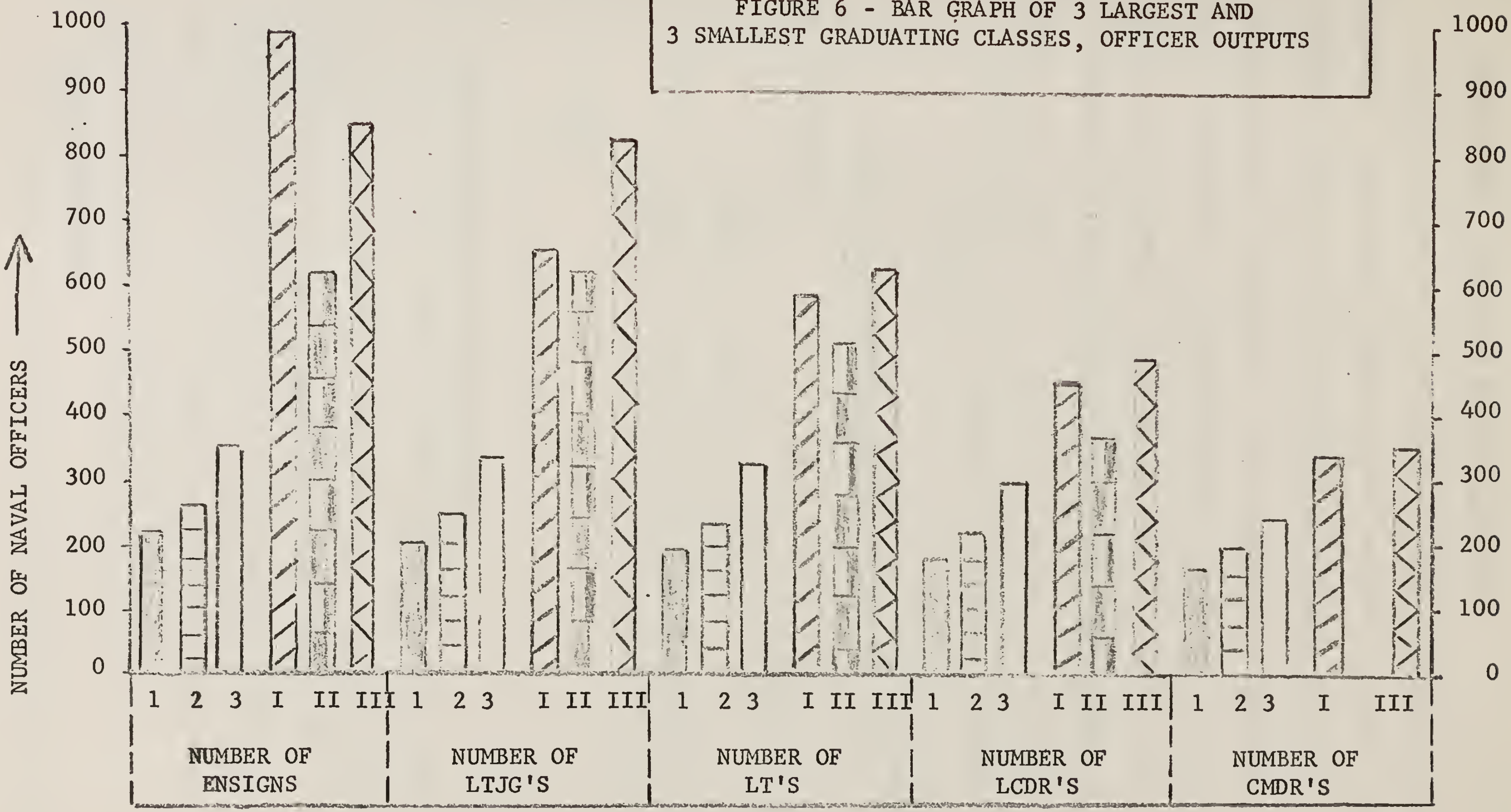


FIGURE 6 - BAR GRAPH OF 3 LARGEST AND 3 SMALLEST GRADUATING CLASSES, OFFICER OUTPUTS

LEGEND:

1	SMALLEST GRADUATING CLASS '36	I	LARGEST GRADUATING CLASS '46
2	SECOND SMALLEST GRADUATING CLASS '37	II	SECOND LARGEST GRADUATING CLASS '53
3	THIRD SMALLEST GRADUATING CLASS '41	III	THIRD LARGEST GRADUATING CLASS '45

A. Basic Model - Repeated Bernoulli Trials

The basic experiment consisted of testing whether or not an Academy graduate, commissioned ENSIGN, served in grade 1(LTJG), 2(LT),...... 6(RADM or above). If so, the trial was a success; if not, a failure. The underlying distribution of the population is BERNOULLI. If it could be empirically determined that the proportion of graduates commissioned ENSIGN in a graduating class for grade i was a constant for several classes, a mathematical implication clearly followed that the classes sampled were indeed from the same population.

Let p_i = population of graduates, commissioned ENSIGN, who served in grade i.

i.e. P(a randomly selected graduate commissioned ENSIGN from any Naval Academy class serves in grade i) = p_i

B. Estimators:

Let N_i = number of graduates commissioned ENSIGN from Naval Academy classes in which p_i = constant

let n_i = number of graduates, commissioned ENSIGN from Naval Academy classes in which p_i = constant, who served in grade i

$$\hat{p}_i = \frac{n_i}{N_i}$$

C. Least Squares Fit of Straight Line:

Data for recent classes relating to the various grades was used to fit (by standard methods) a straight line. Computations are included in Appendix I. Slopes of these trend lines ranged from .017 to .65. These lines approached being horizontal which indicated a nearly constant proportion p_i. It was not known that the data for recent classes would reveal nearly constant proportions of success.

D. Point Estimation:

Point estimation was used in estimating p_j for three situations:

1. $\hat{P}_I = \frac{\text{USNA graduates Commissioned Ensign}}{\text{USNA INPUT of 4th Classmen}}$

$N_j =$

JUSTIFICATION: Large sample size (5570) and slope = .36 for least squares fit.

2. $\hat{P}_{CDR} = \frac{\text{USNA graduates serving in grade of Commander}}{\text{USNA graduates serving in grade of Lieutenant Commander}}$

$N_j =$

JUSTIFICATION: Large sample size (1551) which covers a fifteen year period of service for each success, and slope = -.58 for least squares fit.

3. $\hat{P}_{CAPT} = \frac{\text{USNA graduates serving in grade of Captain}}{\text{USNA graduates serving in grade of Commander}}$

$N_j =$

JUSTIFICATION: Large sample size (943) which covers an eighteen year period for success, and slope = .55 for least square fit.

E. Approximation to Normal Distribution

For large values of N, the BERNOULLI DISTRIBUTION of the population is very closely normally distributed.

F. Sampling Distribution of Proportions $\hat{p}_i$

$$E(\hat{p}_i) = p_i$$

$$VAR\ \hat{p}_i = (N_i)(p_i)(1 - p_i)$$

G. Use of one sided lower confidence limits

Generally, in reliability prediction we are interested not in how good the item could be (how many Captains we might get), but rather in what is the worst that can be expected. Therefore, a lower one sided limit is used throughout.

FORMULA: $$P\left(p_i \geq \hat{p}_i - Z_\alpha \sqrt{\frac{\hat{p}_i(1 - \hat{p}_i)}{N_i}}\right) = \alpha$$

NOTATION: p_i = true proportion for grade i

$\hat{p}_i$ = best estimator for p_i

N_i = sample size for grade i

α = confidence level

Z_α = confidence coefficients for normal curve areas for different confidence limits

such that

α	.95	.90	.85	.80
Z_α	1.645	1.282	1.036	.842

H. Use of Conditional Probability:

Conditional Probability was necessary to ultimately determine estimators $\hat{p}_4$ and $\hat{p}_5$ i.e., ENSIGN to CDR and ENSIGN to CAPTAIN. In Figure 2, "%CDR" and "%CAPT" curves simply did not indicate constant trends or approximations thereto. Looking at this problem and data in another perspective, one can reason as follows: The classes which have in the past ten years or so been promoted to CDR and CAPT underwent attrition as junior officers which was primarily due to the demobilization of the Navy after World War II. However, once committed to the Navy and upon reaching the grades of LCDR and CDR, their attrition in these grades in pursuit of CDR and CAPT is considered valid representation for the populations concerned. When the data for Figure 3 was plotted, the "tails" of the curves in question had stabilized. (See Appendix I)

I. INPUT/OUTPUT Formula:

Derivation of the equation follows:

Let O_i = U. S. Naval Academy OUTPUT of ENSIGNS needed for <u>at least</u> 100 officers of grade i, $\left[i = 1(\text{LTJG}), \ldots\ldots 6(\text{RADM or above}) \right]$, with confidence α.

Let $\hat{p}_i(\alpha)$ = Best estimator of proportion of success for grade i with confidence α.

Then $\left(\hat{p}_i(\alpha)\right) O_i = 100$. Equivalently $O_i = \dfrac{100}{\hat{p}_i(\alpha)}$

Let I = U. S. Naval Academy 4th class INPUT required to produce O_i OUTPUT of ENSIGNS

and let $P_I = \dfrac{O_i}{I}$

In order to estimate the true P_I, a POINT ESTIMATION was made from data for USNA classes 1959 - 1963 inclusive, such that $\hat{P}_I = .590 \doteq P_I$ (See Appendix I)

Now $I = \dfrac{O_i}{P_I}$ and substituting

$$\text{for } O_i = \left(\frac{100}{\hat{p}_i(\alpha)}\right)$$

$$I = \left(\frac{100}{\hat{p}_i(\alpha)}\right)\left(\frac{1}{P_I}\right) \doteq \left(\frac{100}{p_i(\alpha)}\right)\left(\frac{1}{\hat{P}_I}\right)$$

$$I \doteq \left(\frac{100}{p_i(\alpha)}\right)\left(\frac{1}{.590}\right) \doteq \left(\frac{169.4}{p_i(\alpha)}\right)$$

This equation will compute USNA inputs required to assure the Navy of at least 100 officers of grade i, with confidence α. (Or by fixing I, O_i's can be computed.)

In using the point estimation method for estimating the probability of a randomly selected 4th classman to be graduated and commissioned ENSIGN, an approximation is introduced into the equation. It is pointed out that this statistical estimating relationship is not the ultimate. Results should be tempered by judgement, experience, and information relevant to the analysis at hand. Obviously one constraint is the physical limitation of the Naval Academy itself.

APPENDIX I

STATISTICAL ANALYSIS AND COMPUTATIONS

I Regression Analysis

For each proportion $\hat{p}_i$ a simple regression analysis has been performed. Standard notation is as follows:

X = U. S. Naval Academy class (year) which is variable

$$\bar{X} = \frac{\sum_{i=1}^{n} X}{n} \quad ; \qquad n+1 = \text{number of classes sampled}$$

X = 0 is the ORIGIN for the FIRST class sampled, and the units of X are 1 year.

$$x = X - \bar{X}$$

Y = Percentage of success for the class used in the sample, which is variable

$$\bar{Y} = \frac{\sum_{i=1}^{n} Y}{n} \qquad \text{and } y = Y - \bar{Y}$$

The regression curve of Y on X is in effect a trend line for the time series and as will be shown, it is reasonably constant, i.e., horizontal line. $Y = a_0 + a_1 X$, where a_1 approaches zero.

a. Regression analysis of USNA inputs and ENSIGN OUTPUTS, per class

1. From Table VII

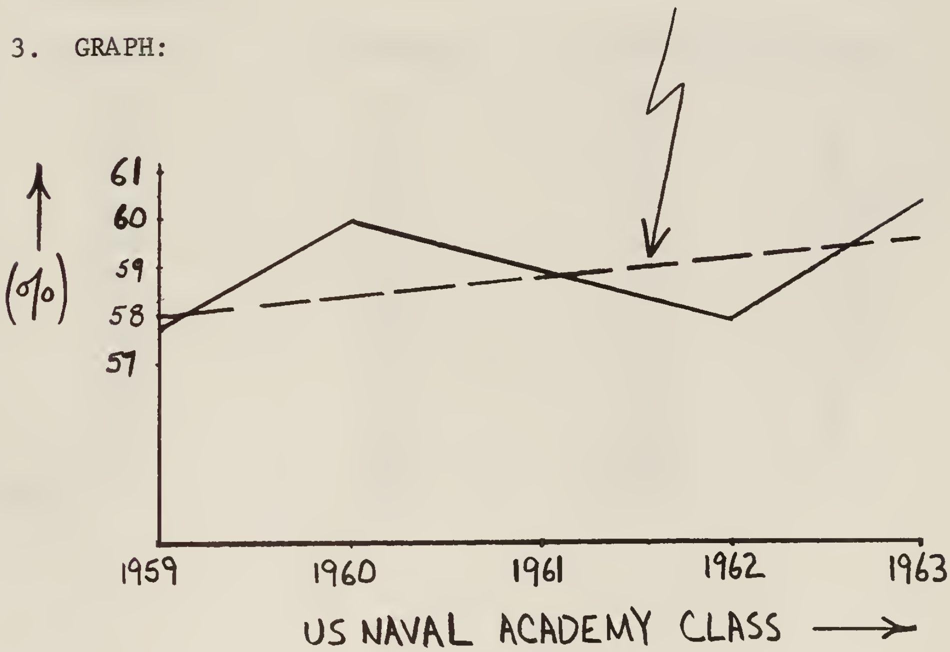
NA CLASS	NR ENTERED	# ENSIGNS	(%) ENS INPUT
1959	1102	638	57.7
1960	1075	648	60.0
1961	1126	664	59.0
1962	1074	624	58.0
1963	1193	722	60.5

2. LEAST SQUARES FIT OF STRAIGHT LINE

	X	Y	$x=X-\bar{X}$	$y=Y-\bar{Y}$	x^2	xy
1959	0	57.7	-2	-1.34	4	+2.68
1960	1	60.0	-1	+ .96	1	- .96
1961	2	59.0	0	- .04	0	0
1962	3	58.0	+1	+1.04	1	-1.04
1963	4	60.5	+2	+1.46	4	+2.92
	$\bar{X}=2$	$\bar{Y}=59.04$			10	+3.60

Equation of fit $Y = 58.32 + .36X$

3. GRAPH:



4. <u>Statistical Inference</u>:

let P_I = proportion of entering 4th class that are commissioned ENSIGN

$E\left[P_I / \text{class year}\right] \doteq$ CONSTANT

$\longrightarrow$ these classes are in the same population

i.e. $P\left[\text{a randomly selected 4th classman from any of these 5 classes is commissioned ENSIGN}\right] = P_I$

A <u>point estimate</u> of the true $P_I \doteq \hat{P}_I = \dfrac{\text{NR of ENSIGNS from all 5 classes}}{\text{NR of 4th classmen from all 5 classes}}$

$$\hat{P}_I = \frac{638+648+664+624+722}{1102+1075+1126+1074+1193} = \frac{3296}{5570} = .590$$

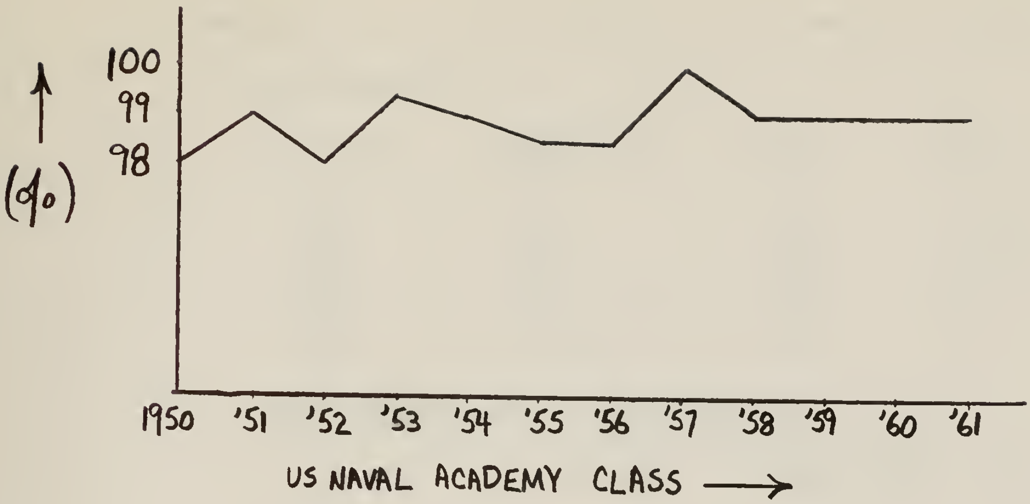
5. Since the true P_I is never known and considering the large sample size, $\hat{P}_I = .590$ is a valid POINT ESTIMATE.

b. Regression Analysis of ENSIGN to Lieutenant(jg), per class

1. <u>From Table V</u>:

NA. CLASS	COMM ENS	# LTJG	% LTJG
1950	462	454	98
1951	483	477	99
1952	514	507	98
1953	619	616	99.4
1954	559	552	99
1955	495	489	98.5
1956	454	449	98.5
1957	567	567	100
1958	624	617	99
1959	638	635	99
1960	648	644	99
1961	664	663	99
TOTALS	6727	6670	

2. <u>GRAPH</u>:



a. Least squares fit not necessary

3. <u>Inference</u>:

$$\hat{p}_i = \frac{m_1}{N_1} = \frac{6670}{6727} = .991$$

4. One sided lower confidence limits are computed as follows:

a. The population sampled is BERNOULLI.

b. Since N is large $\longrightarrow$ Distribution closely approximates the NORMAL DISTRIBUTION.

c. $P\left(p_1 \geq \hat{p}_1 - Z_\alpha \sqrt{\frac{\hat{p}_1 (1 - \hat{p}_1)}{N_1}}\right) = \alpha$

(1) $P\left(p_1 \geq .991 - 1.645 \sqrt{\frac{(.991)(.009)}{6727}}\right) = .95$

$P(p_1 \geq .9891) = .95$

(2) $P(p_1 \geq .9895) = .90$

(3) $P(p_1 \geq .9898) = .85$

(4) $P(p_1 \geq .99) = .80$

c. Regression Analysis of ENSIGN to Lieutenant, per class

1. From Table V:

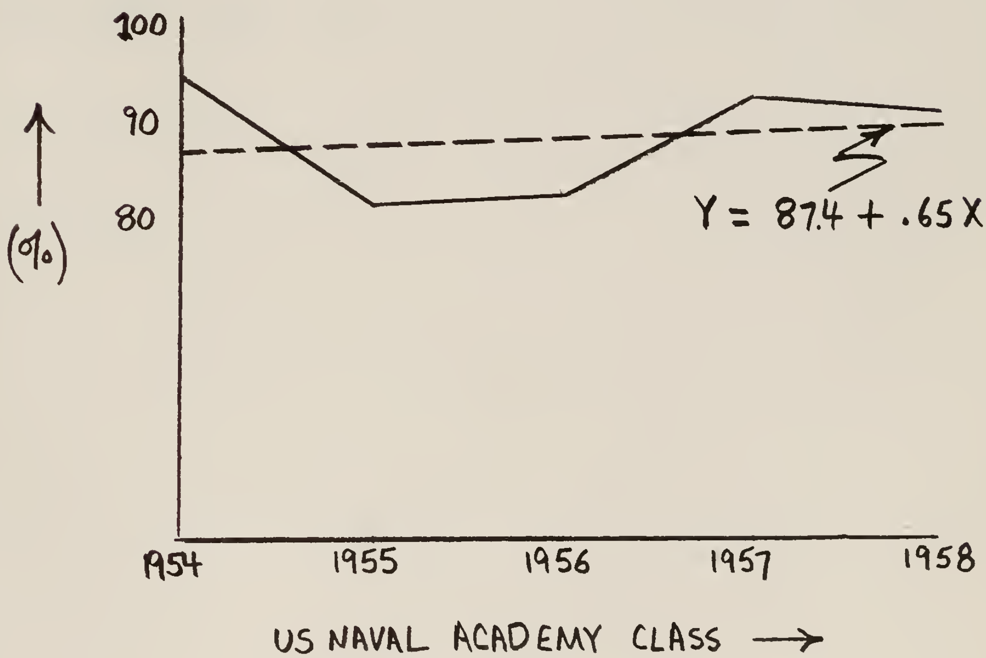
NA CLASS	COMM. ENS	# LT	% LT
1954	559	528	94.5
1955	495	404	81.5
1956	454	375	82.5
1957	567	529	93
1958	624	573	92

2. Least Squares Fit of Straight Line:

	X	Y	$x = X - \bar{X}$	$y = Y - \bar{Y}$	x^2	xy
1954	0	94.5	-2	+5.8	4	-11.6
1955	1	81.5	-1	-7.2	1	+ 7.2
1956	2	82.5	0	-6.2	0	0
1957	3	93	1	+4.3	1	+ 4.3
1958	4	92	2	+3.3	4	+ 6.6
	10	443.5			10	+ 6.5

$\bar{X}=2$ $\bar{Y}=88.7$

3. Graph



4. Inference:

$$\hat{p}_2 = \frac{m_2}{N_2} = \frac{528+404+375+529+573}{559+495+454+569+624} = \frac{2409}{2699} = .896$$

5. Confidence Limits:

$$P\left(p_2 \geq \hat{p}_2 - Z_\alpha \sqrt{\frac{\hat{p}_2\ (1-\hat{p}_2)}{N_2}}\right) = \alpha$$

(1) $P\left(p_2 \geq .896 - 1.645 \sqrt{\frac{(.896)(.104)}{2699}}\right) = .95$

$P(p_2 \geq .886) = .95$

(2) $P(p_2 \geq .888) = .90$

(3) $P(p_2 \geq .889) = .85$

(4) $P(p_2 \geq .891) = .80$

d. Regression Analysis of ENSIGN to LCDR

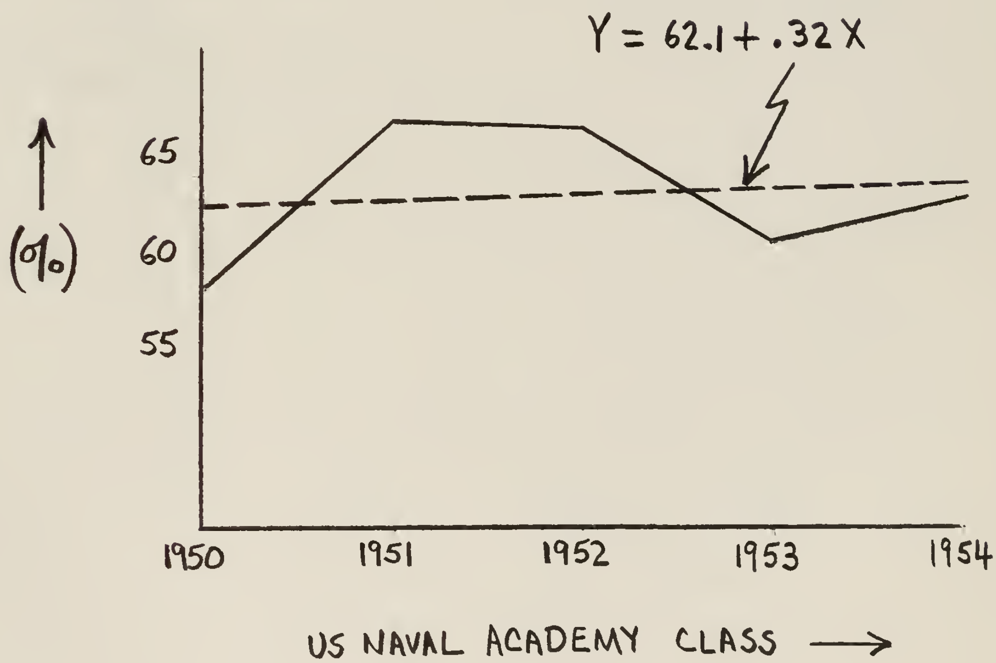
1. From Table V:

NA CLASS	COMM ENS	# LCDR	% LCDR
1950	462	268	58
1951	483	320	66.4
1952	514	340	66.1
1953	619	374	60.4
1954	559	350	62.6

2. Least Squares Fit of Straight Line

	X	Y	$x = X-\bar{X}$	$y=Y-\bar{Y}$	x^2	xy
1950	0	58	-2	-4.7	4	+9.4
1951	1	66.4	-1	+3.7	1	-3.7
1952	2	66.1	0	+3.4	0	0
1953	3	60.4	1	-2.3	1	-2.3
1954	4	62.6	2	-.1	4	- .2
	$\bar{X}=2$	$\bar{Y}=62.7$			10	+3.2

3. GRAPH:



4. <u>Inference</u>:

$$\hat{p}_3 = \frac{m_3}{N_3} = \frac{268+320+340+374+350}{462+483+514+619+559} = \frac{1652}{2637} = .629$$

5. <u>Confidence Limits</u>:

$$P\left(p_3 \geq \hat{p}_3 - Z_\alpha \sqrt{\frac{\hat{p}_3\ (1 - \hat{p}_3)}{N_3}}\right) = \alpha$$

(1) $$P\left(p_3 \geq .629 - 1.645 \sqrt{\frac{(.629)(.371)}{2637}}\right) = .95$$

$$P(p_3 \geq .614) = .95$$

(2) $P(p_3 \geq .617) = .90$

(3) $P(p_3 \geq .619) = .85$

(4) $P(p_3 \geq .621) = .80$

e. Regression Analysis of ENSIGN to COMMANDER

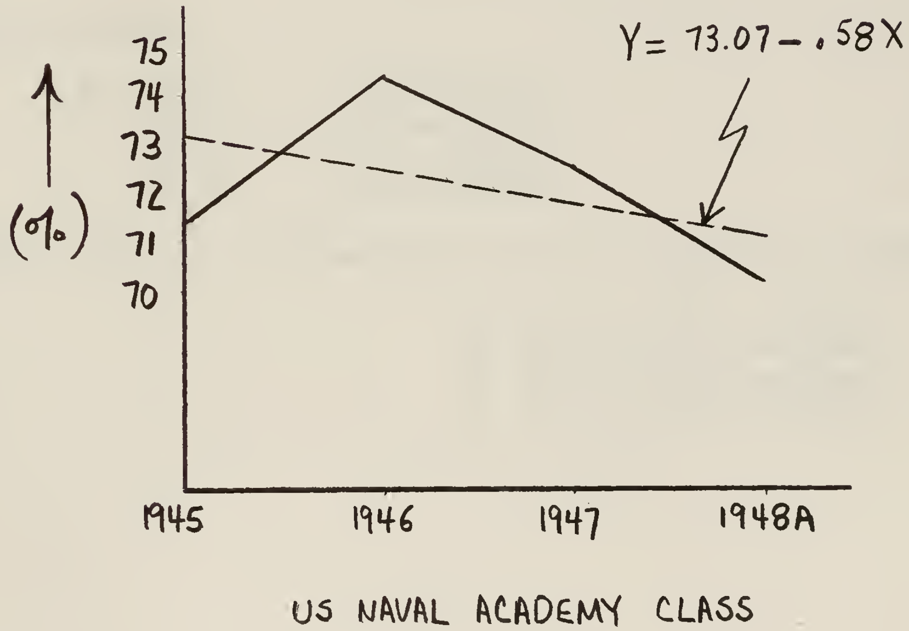
1. Data from Table VI:

USNA CLASS	# LCDR	# CMDR	$\frac{\text{CMDR}}{\text{LCDR}}$ %
1945	497	355	71.4
1946	456	341	74.6
1947	364	265	72.7
1948A	234	164	70.1

2. Least Squares Fit of Straight Line:

USNA CLASS	X	Y	$x = X-\bar{X}$	$y = Y-\bar{Y}$	x^2	xy
1945	0	71.4	-1.5	-.80	2.25	+1.2
1946	1	74.6	- .5	+2.4	.25	-1.2
1947	2	72.7	+ .5	+ .5	.25	+ .25
1948A	3	70.1	+1.5	-2.1	2.25	-3.15
					5.0	-2.9

3. Graph



4. <u>Inferences</u>:

$P_{CMDR} = \frac{1125}{1551} = .724$ = POINT ESTIMATE of probability of serving in grade of CMDR, given the grade of LCDR

5. <u>Conditional Probability</u>

Let D = event ENS serves as a CMDR

Let C = event ENS serves as a LCDR

Let D/C = event ENS serves as a CMDR, given ENS serves as LCDR

$P[D/C] \doteq \hat{P}_{CMDR}$ (POINT ESTIMATE) = .724

$P[D/C] = \frac{P[DC]}{P[C]}$ BUT $P[DC] = P[D]$

$P[D/C] = \frac{P[D]}{P[C]}$; $P[D] = P[D/C]\ P[C]$

$P[D] \doteq (.724)\ P[C]$

But from section d

$$
\begin{aligned}
P[C] &\geq .614 \quad \text{, 95\% confidence} \\
&\geq .617 \quad \text{, 90\% confidence} \\
&\geq .619 \quad \text{, 85\% confidence} \\
&\geq .621 \quad \text{, 80\% confidence}
\end{aligned}
$$

$\Longrightarrow$ $P[D]$ is approximately $\geq$ (.614)(.724) with 95% confidence

or

$\geq$ .444 with 95% confidence

$\geq$ .446 with 90% confidence

$\geq$ .448 with 85% confidence

$\geq$ .450 with 80% confidence

f. Regression Analysis of ENSIGN to Captain

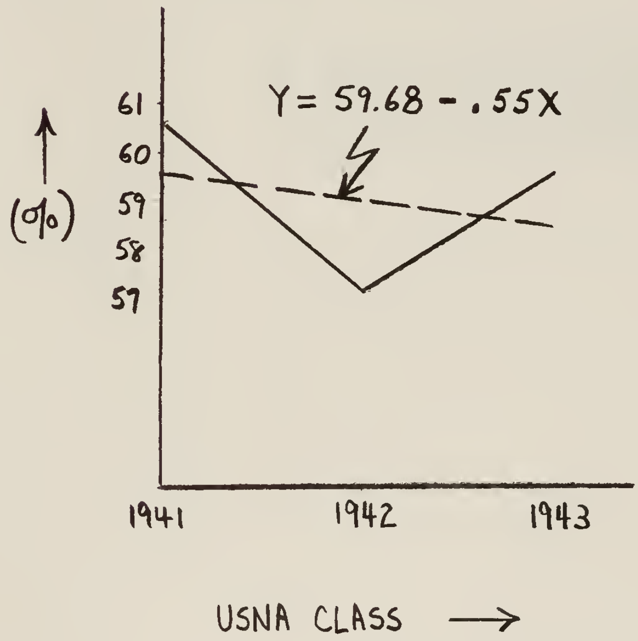
1. Data from Table VI:

USNA CLASS	NR CMDR	NR CAPTAIN	CAPT / % CMDR
1941	245	149	60.7
1942	336	192	57.1
1943	362	216	59.6

2. Least Squares Fit of Straight Line

USNA CLASS	X	Y	$x = X-\bar{X}$	$y = Y-\bar{Y}$	x^2	xy
1941	0	60.7	-1	+1.57	1	-1.57
1942	1	57.1	0	-2.03	0	0
1943	2	59.6	+1	+ .47	1	+ .47
					2	-1.10

3. GRAPH:



4. Inference:

$\hat{P}_{CAPT} = \frac{557}{943} = .591$ = POINT ESTIMATE of probability of serving in the grade of CAPTAIN, given already serving in the grade of Commander

5. Conditional Probability

Using identical method used in preceeding section e

$$P\left[\text{CAPTAIN}\right] \doteq (.591)\ P\left[\text{COMMANDER}\right]$$

But from section e

$$
\begin{aligned}
P\left[\text{COMMANDER}\right] &\geq .444 \text{ with } 95\% \text{ confidence} \\
&\geq .446 \text{ with } 90\% \text{ confidence} \\
&\geq .448 \text{ with } 85\ \% \text{ confidence} \\
&\geq .450 \text{ with } 80\% \text{ confidence}
\end{aligned}
$$

$$\Longrightarrow P\left[\text{ENSIGN graduate serving in grade CAPTAIN}\right] \geq (.591)(.444),$$

with 95% confidence

$$
\begin{aligned}
&\geq .262 \text{ with } 95\% \text{ confidence} \\
&\geq .264 \text{ with } 90\% \text{ confidence} \\
&\geq .265 \text{ with } 85\% \text{ confidence} \\
&\geq .266 \text{ with } 80\% \text{ confidence}
\end{aligned}
$$

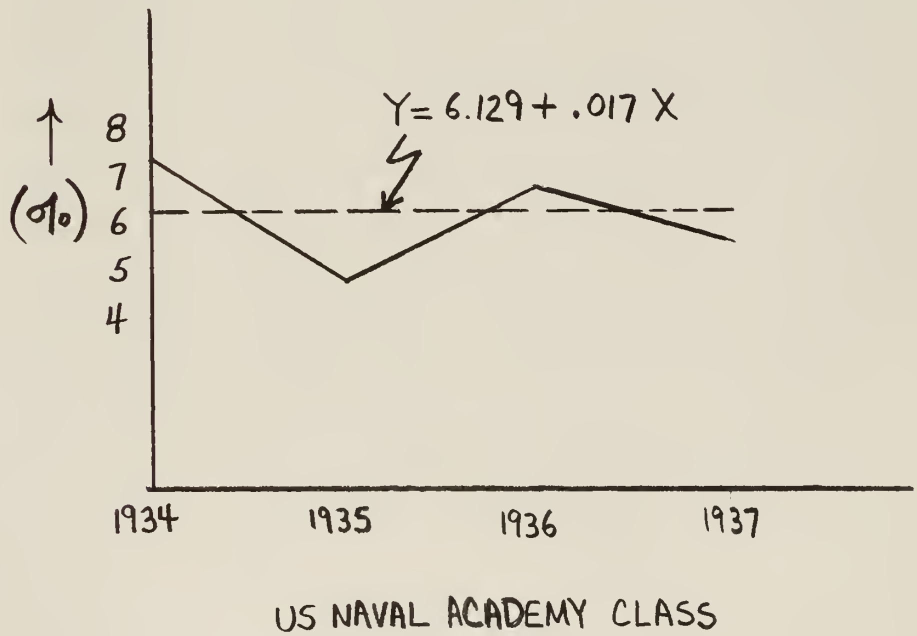
g. Regression Analysis of ENSIGN to REAR ADMIRAL (or above)

	USNA CLASS	NR ENS	NR FLAG OFFICERS	% RADM
	1934	330	24	7.28
	1935	369	18	4.87
	1936	219	15	6.84
	1937	266	15	5.63
	1938	386	12 (incomplete)	
	1939	504	4 (incomplete)	
TOTALS		1184	72	

2. Least Squares Fit of Straight Line

USNA CLASS	X	Y	$x = X - \bar{X}$	$y = Y - \bar{Y}$	x^2	xy
1934	0	7.28	-1.5	+1.125	2.25	-1.6875
1935	1	4.87	- .5	-1.285	.25	+ .6425
1936	2	6.84	+ .5	+ .685	.25	+ .3425
1937	3	5.63	+1.5	+ .525	2.25	+ .7875
					5.00	+ .0850

3. GRAPH:



4. <u>Inference</u>:

$$\hat{P}_6 = \frac{72}{1184} = .0607$$

5. One Sided Lower Confidence Limits

Formula: $P\left(p_6 \geq \hat{p}_6 - Z_\alpha \sqrt{\frac{\hat{p}_6(1-\hat{p}_6)}{N_6}}\right) = \alpha$

a. $P(p_6 \geq .0247) = .95$

b. $P(p_6 \geq .0325) = .90$

c. $P(p_6 \geq .0380) = .85$

d. $P(p_6 \geq .0422) = .80$

II INPUT/OUTPUT Computations:

1. The basic equation for this computation was developed in part I, section 5.

$$\text{INPUT} = \frac{O}{P_I} = \left(\frac{100}{p_i(\alpha)}\right)\left(\frac{1}{P_I}\right)$$

p_i (.95) values are simply those computed and are listed in Table 1, and

$P_I \doteq .590$

a. Given: OUTPUT 100 LTJG required (95% confidence)

ENS OUTPUT = $\frac{100}{.9891}$ = 101; INPUT = $\frac{101}{.59}$ = 171 4th classmen

LT OUTPUT = (101)(.886) = 90
LCDR OUTPUT = (101)(.614) = 62
CMDR OUTPUT = (101)(.444) = 45
CAPT OUTPUT = (101)(.262) = 26
RADM OUTPUT = (101)(.0247)= 2

b. Given: OUTPUT 100 LT's required (95% confidence)

ENS OUTPUT = $\frac{100}{.886}$ = 113 ; INPUT = $\frac{113}{.59}$ = 191

LTJG OUTPUT = (113)(.9891) = 111
LCDR OUTPUT = (113)(.614) = 69
CMDR OUTPUT = (113)(.444) = 50
CAPT OUTPUT = (113)(.262) = 29
RADM OUTPUT = (113)(.0247) = 3

c. Given: OUTPUT 100 LCDR's required (95% confidence)

ENS OUTPUT = $\frac{100}{.614}$ = 163 ; INPUT = $\frac{163}{.59}$ = 276

LTJG OUTPUT = (163)(.9891) = 161
LT OUTPUT = (163)(.886) = 144
CMDR OUTPUT = (163)(.444) = 72
CAPT OUTPUT = (163)(.262) = 43
RADM OUTPUT = (163)(.0247) = 4

d. <u>Given</u>: OUTPUT 100 CMDR's required (95% confidence)

$$\text{ENS OUTPUT} = \frac{100}{.444} = 225 \; ; \; \text{INPUT} = \frac{225}{.59} = 382$$

LTJG OUTPUT = (225)(.9891) = 222
LT OUTPUT = (225)(.886) = 200
LCDR OUTPUT = (225)(.614) = 138
CAPT OUTPUT = (225)(.262) = 59
RADM OUTPUT = (225)(.0247) = 6

e. <u>Given</u>: OUTPUT 100 Captain's required (95% confidence)

$$\text{ENS OUTPUT} = \frac{100}{.262} = 382 \; ; \; \text{INPUT} = \frac{382}{.59} = 646$$

LTJG OUTPUT = (382)(.9891) = 377
LT OUTPUT = (382)(.886) = 339
LCDR OUTPUT = (382)(.614) = 234
CMDR OUTPUT = (382)(.444) = 170
RADM OUTPUT = (382)(.0247) = 9

2. This method may be used to compute any set of INPUTS/OUTPUTS served, at a specified confidence, <u>given an input</u>, <u>or an output</u>.